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Portfolio Optimization Using Matrix Approach: A Case of Some Stocks on the Ghana Stock Exchange

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Abstract: Analyzing risk has been a principal concern of actuarial and insurance professionals which plays a fundamental role in the theory of portfolio selection where the prime objective is to find a portfolio that maximizes expected return while reducing risk. Portfolio optimization has been applied to asset management and in building strategic asset allocation. The purpose of this paper is to construct optimal and efficient portfolios using the matrix approach. This paper used secondary data on 13 stocks (ETI, GCB, GOIL, TOTAL, FML, GGBL, CLYD, EGL, PZC, UNIL, TLW, AGA and BOPP) from the Ghana Stock Exchange (GSE) database comprising the monthly closing prices from the period 02/01/2004 to 16/01/2015. The results revealed that, all the portfolios were optimal and that portfolios 1, 2, 4, 5, 6, 9, 10, 11 and 12 with expected return 2.523, 2.593, 2.827, 3.642, 2.405, 2.812, 5.229, 3.559 and 5.928 respectively were efficient portfolios whereas portfolios 3, 7 and 8 with expected return 0.377, 0.699 and 0.152 respectively were inefficient portfolios with reference to the expected return of the global minimum variance portfolio (2.360). GGBL was seen as the stock with the highest allocation of wealth in most of the portfolios. Six out of the 12 portfolios had CLYD exhibiting the least asset allocation.

Keywords: Portfolio Optimization, Efficient Frontier, Mean-Variance, Matrix Approach

1. Introduction

After Markowitz ground-breaking work in portfolio selection Markowitz [10] portfolio optimization has been receiving greater attention from asset and liability managers, academics and risk managers. Most of the studies explain a portfolio optimization criterion such as mean-variance, conditional value-at-risk, value-at-risk, mean absolute deviation, stochastic dominance of first and second order among others.

The mean-variance is the traditional optimization approach introduced by Markowitz. But before Markowitz presented his approach, portfolio theory was a relevant area of research. However, the main focus of Bachelier and his successor was to improve performance. Markowitz focused on risk. He established volatility as a major risk measure in portfolio theory and showed how the risk can be reduced by diversification. He demonstrated how financial portfolios which have max expected return for a given risk level can be estimated.

In portfolio analysis, variance measures the volatility (risk) of an asset or group of assets, hence larger variance indicates greater risk and vice versa. When many assets are held together in a portfolio, assets decreasing in value are usually offset by portfolios asset increasing in value, hence minimizing risk. Also, the total variance of a portfolio is usually lower than a simple weighted average of the individual asset variances [5]. The return of any financial asset is described by a random variable, whose expected mean and variance are assumed to be reliably estimated from historical data. The expected mean and variance are interpreted as the compensation and the risk respectively. The portfolio optimization problem can be formulated as follows: given a set of assets, characterized by their returns and covariance, find the optimal weight of the asset such that the overall portfolio provides the lowest risk for a given overall return. This problem reduces to find the efficient frontier, which is the set of all achievable (attainable) portfolios that offers a higher return for a given risk level. When the number of assets in a portfolio becomes large, the total variance is actually derived from the covariance than from the variances of the assets [15].

The theory of portfolio optimization is generally associated with the classical mean-variance optimization framework of Markowitz [10]. The drawback of the mean-variance analysis is mainly related to its sensitivity to the estimation error of the means and covariance matrix estimation of the returns of the asset. Also, it is argued that estimates of the covariance matrix are more accurate than those of the expected returns ([12], [8]). Several studies concentrates on improving the performance of the global minimum-variance portfolio (GMVP), which provides the least possible portfolio risk and involves only the covariance matrix estimates.

The classical mean-variance framework depends on the perfect knowledge of the expected returns of the assets and their variance-covariance matrix. However, these returns are unobservable and unknown. The impossibility to obtain a sufficient number of data samples, instability of data, differing personal views of decision makers on the future returns [13] affect their estimation and has led to what [1] call estimation risk in portfolio selection. This estimation risk has shown to be the source of very erroneous decisions, for, as pointed in ([2], [6]), the composition of the optimal portfolio is very sensitive to the mean and the covariance matrix of the asset returns and agitation in the moments of the random returns can result in the difficulties in constructing different optimization.

[9], examined portfolio optimization with correlation matrix. The results showed how to perform portfolio optimizations using mean-correlation instead of mean-variance analysis and that the two alternatives set-up produced equivalent optimization weights if correlation-based number transformed back to mean-variance ones. Also, the analysis, presented strengthens the role of regression methods in portfolio analysis. [14], presented a simplified perspective of Markowitz contributions to Modern Portfolio Theory (MPT), foregoing in-depth presentation of the complex mathematical/statistical models typically associated with discussions of this theory and suggested efficient computer-based 'short cuts'.

Also, ([3], [16], [7]) have studied the mean variance framework in a robust context, assuming that the expected return is stochastic. They characterize the parameters involved in the mean and the variance-covariance matrix with specific types of uncertainty, and built semi-definite or second-order cone programs.

On efficient and optimal portfolios, [4] stated that portfolios are efficient when they provide the maximum possible expected return for a certain risk level. When building efficient portfolio one need to assume that investors are risk-averse, meaning that they will choose the portfolio with the least risk. When faced with several portfolios with the same expected return, but with different risk levels. Also, a risk-averse investor will choose the portfolio with the highest return, when they have to choose from a set of portfolios with the same risk, but different expected returns. This indicates that efficient portfolios are located in the efficient frontier (minimum-variance frontier). An optimal portfolio is one that has the minimum risk for a given level of return and an efficient portfolio is one that has the maximum expected rerun for a given level of risk. Thus, all portfolios on the minimum-variance frontier are optimal, but only those in the upper portion-at above the global minimum-variance portfolio are efficient.

The purpose of this paper is to construct optimal and efficient portfolios using matrix approach. This will give investors an insight in diversification, asset management and risk management. It will also aid investors and academics on how to construct optimal and efficient portfolios using matrix approach.

2. Materials and Methods

2.1. Source of Data

This paper used secondary data of 13 stocks (ETI, GCB, GOIL, TOTAL, FML, GGBL, CLYD, EGL, PZC, UNIL, TLW, AGA and BOPP) from the Ghana Stock Exchange (GSE) database comprising the daily closing prices from the period 02/01/2004 to 16/01/2015.

2.2. Methods of Data Analysis

The daily index series were converted into compound returns given by;

$$R_t = \log\left(\frac{p_t}{p_{t-1}}\right) \tag{1}$$

where R_t is the continuous compound returns at time t, p_t is the current closing stock price index at time t and p_{t-1} is the previous closing stock price index. These returns were converted into monthly returns by assuming 365 days a year and averaging to get 30 days a month. This was then multiplied by the daily returns to obtain the monthly returns. The same method was employed in obtaining the monthly standard deviations by multiplying the square root of 30 by the daily standard deviations.

For an n-asset portfolio problem with assets given by $1,2,\dots,n$. Let $R_i = (i = 1,2,\dots,n)$ denote the return on asset i with a constant expected return model given by

$$R_i \sim iidN(\mu_i, \sigma_i^2) \tag{2}$$

$$Cov(R_i, R_j) = \sigma_{ij} \tag{3}$$

Assuming that all wealth in the n-asset is given by

$$w_1 + w_2 + \cdots + w_n = 1 \tag{4}$$

Then, the portfolio return, $R_{p,w}$ is given by

$$R_{p,w} = w_1R_1 + w_2R_2 + \cdots + w_nR_n \tag{5}$$

where $w_1, w_2, \dots, w_n$ are the weights of the portfolio and $R_1, R_2, \dots, R_n$ are the returns of the individual stocks.

From Equation 5, the expected return on the portfolio is given by

$$\mu_{p,w} = E\left[R_{p,w}\right] = w_1\mu_1 + w_2\mu_2 + \cdots + w_n\mu_n \tag{6}$$

and the variance of the portfolio return given by

$$\sigma_{p,w}^2 = var\left(R_{p,w}\right) \tag{7}$$

2.3. Portfolio Characteristics Using Matrix Approach

The asset returns and portfolio weights are given by an $n \times 1$ column vector;

$$R = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{pmatrix} \tag{8}$$

$$W = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix};\ \sum_{i=1}^{n} w_i = 1 \tag{9}$$

The probability distribution of R is the joint distribution of the elements of R. In the constant expected model, all returns are jointly normally distributed and is characterized by the mean, variance and covariance of the returns. Applying matrix notation, the $n \times 1$ vector of portfolio expected return is given by

$$E[R] = E\left[\begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{pmatrix}\right] = \begin{pmatrix} E[R_1] \\ E[R_2] \\ \vdots \\ E[R_n] \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} = \mu \tag{10}$$

and the $n \times n$ covariance matrix of returns; $C_{ij} = Cov\left(R_i, R_j\right)$ is given by

$$C = \begin{pmatrix} c_{11} & c_{12} \dots & c_{1n} \\ c_{21} & c_{22} \dots & c_{2n} \\ \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & c_{nn} \end{pmatrix} = \Sigma \tag{11}$$

$$Var(R) = Var\left(\sum_{i=1}^{n} w_i\, R_i\right) = Cov\left(\sum_{i=1}^{n} w_i\, R_i, \sum_{i=1}^{n} w_j\, R_j\right) = \sum_{i=j=1}^{n} w_i w_j\, c_{ij} = wCw' \tag{12}$$

Also, the condition that the portfolio weights sum to one (1) is given by

$$w'\varepsilon = (w_1, w_2, \dots, w_n).\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = w_1, w_2, \dots, w_n = 1 \tag{13}$$

where ε is an $n \times 1$ vector with entries equal 1.

2.4. Estimating the Global Minimum Variance Portfolio

The global minimum-variance portfolio is simply the portfolio on the efficient frontier that has the least risk. It is given by

$$m = (m_1, m_2, \dots, m_n)' \tag{14}$$

where $m_1, m_2, \dots, m_n$ are the global minimum-variance portfolio weights for asset$1, 2, \dots, n$.

For an $n-$ asset case, the constrained minimization problem is given by

$$\min_{m_1, m_2, \dots, m_n} \sigma_{p,m}^2 = m_1^2\sigma_1^2 + m_2^2\sigma_2^2 + \cdots + m_n^2\sigma_n^2 + 2m_1m_2\sigma_{12} + 2m_1m_n\sigma_{1n} + \cdots + 2m_{n-1}m_n\sigma_{n(n-1)} \tag{15}$$

Thus,

$$m_1 + m_2 + \cdots + m_n = 1 \tag{16}$$

Also, the first order linear equation is given by

$$\begin{pmatrix} 2\sigma_1^2 & 2\sigma_{12} \dots & 2\sigma_{1n} & 1 \\ 2\sigma_{21} & 2\sigma_2^2 \dots & 2\sigma_{2n} & 1 \\ 2\sigma_{1n} & 2\sigma_{2n} \dots & 2\sigma_n^2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \lambda \end{pmatrix} \tag{17}$$

Therefore,

$$\begin{pmatrix} 2\Sigma & 1 \\ 1' & 0 \end{pmatrix} \begin{pmatrix} m \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{18}$$

Equation (18) is of the form

$$B_m z_m = k \tag{19}$$

where $B_m = \begin{pmatrix} 2\Sigma & 1 \\ 1' & 0 \end{pmatrix}$, $z_m = \begin{pmatrix} m \\ \lambda \end{pmatrix}$ and $k = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Thus, solving for Z_m from Equation (19), we get

$$z_m = B_m^{-1}k \tag{20}$$

where the elements of z_m are the portfolio weights $m = (m_1, m_2, \dots, m_n)'$ for the global minimum variance portfolio return, $\mu_{p,m} = m'\mu$ and variance, $\sigma_{p,m}^2 = m'\Sigma\text{m}$.

2.5. Determining the Efficient Portfolios

For an n-asset case, the investment opportunity set in $\left(\mu_p, \sigma_p\right)$-space is explained by set of values whose shape depends on the covariance terms. Assuming that investors select portfolios that maximizes expected return subject to a target level of risk or minimize risk subject to a target expected return, the asset allocation problem can be streamlined by only concentrating on the set of efficient portfolios. These portfolios lie on the boundary of the investment opportunity set above the global minimum variance portfolio.

Following Markowitz [11], we assume that investors wish to find portfolios that have the best expected return-risk trade off. Firstly, investors wish to find portfolios that maximizes portfolio expected return for a given risk level as measured by portfolio variance or standard deviation. This constrained maximization problem to find an efficient portfolio is given by

$$\max_w \mu_p = w'\mu \tag{21}$$

$$\sigma_p^2 = w'\Sigma \mathrm{w} = \sigma_{p,o}^2; w'1 = 1 \tag{22}$$

Markowitz also showed that the investor's problem of maximizing portfolio expected return subject to a target risk level has a correspondent dual representation in which the investor minimizes the risk of the portfolio subject to a given expected return. This dual problem is the constrained minimization problem which is given by

$$\min_w \sigma_{p,w}^2 = w'\Sigma \mathrm{w} \tag{23}$$

$$\mu_p = w'\mu = \mu_{p,o}; w'1 = 1 \tag{24}$$

In this paper, the dual problem is considered due to computational convenience and that investors being more willing to specify target expected returns rather than risk.

In solving the constrained minimization problem in Equations (23 and 24), the Lagrangian function is employed and is given by

$$L(w, \lambda_1, \lambda_2) = w'\Sigma \mathrm{w} + \lambda_1\left(w'\mu - \mu_{p,o}\right) + \lambda_2(w'1 - 1) \tag{25}$$

where $w'\mu = \mu_{p,o}$ and $w'1 = 1$ are the two constraints and λ_1 and λ_2 are the two Lagrangian multipliers.

The first order conditions for a minimum are given by the following linear equations

$$\frac{\partial L(w,\lambda_1,\lambda_2)}{\partial w} = 2\Sigma \mathrm{w} + \lambda_1\mu + \lambda_2 1 = 0 \tag{26}$$

$$\frac{\partial L(w,\lambda_1,\lambda_2)}{\partial \lambda_1} = w'\mu - \mu_{p,o} = 0 \tag{27}$$

$$\frac{\partial L(w,\lambda_1,\lambda_2)}{\partial \lambda_2} = w'1 - 1 = 0 \tag{28}$$

Also, representing the system of linear equations in matrix form, we get

$$\begin{pmatrix} 2\Sigma & \mu & 1 \\ \mu' & 0 & 0 \\ 1' & 0 & 0 \end{pmatrix} \begin{pmatrix} w \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \mu_{p,o} \\ 1 \end{pmatrix} \tag{29}$$

Equation (29) is of the form

$$Bz_w = k_o \tag{30}$$

where $B = \begin{pmatrix} 2\Sigma & \mu & 1 \\ \mu' & 0 & 0 \\ 1' & 0 & 0 \end{pmatrix}$, $z_w = \begin{pmatrix} w \\ \lambda_1 \\ \lambda_2 \end{pmatrix}$ and $k_o = \begin{pmatrix} 0 \\ \mu_{p,o} \\ 1 \end{pmatrix}$

Solving for z_w, we get

$$z_w = B^{-1}k_o \tag{31}$$

If $\mu_{p,o} \geq \mu_{p,m}$ then portfolio, $w = (w_1, w_2, \ldots, w_n)'$ is an efficient portfolio otherwise w is an inefficient portfolio. Also, all portfolios on the minimum variance frontier are optimal, but only those in the upper portion (at or above) the global minimum-variance portfolio are efficient.

3. Results and Discussion

Table 1 shows the descriptive statistics of the stocks. With much emphasis on the monthly returns and standard deviations, the results show that, the monthly expected return ranges from -0.547 to 5.928 with the highest return found in BOPP and the least return found in CLYD. All the stocks made gains (positive expected return) with the exception of CLYD which made a loss (negative expected return). The monthly standard deviation (risk) ranged from 8.498 to 35.547 with ETI (35.547) been the stock with the highest risk level compared with GGBL (8.498) which had the least risk level. Even though, the highest mean return was found in BOPP, its risk level was less than that of ETI. This implies that, ETI is much riskier than the rest of the stocks and that an investors need to reduce this risk by diversifying. For risk averse investors, it will be prudent to go in for GGBL since it had the least risk level compared to the rest of the stocks.

Table 1. *Descriptive Statistics.*

Stock	Daily Expected Return	Monthly Expected return	Daily Std. Dev	Monthly Std. Dev
ETI	0.083	2.523	6.463	35.547
GCB	0.085	2.593	1.926	10.594
GOIL	0.012	0.377	2.097	11.536
TOTAL	0.093	2.827	4.328	23.804
FML	0.120	3.642	2.157	11.864
GGBL	0.079	2.405	1.545	8.498
CLYD	-0.018	-0.547	4.598	25.289
EGL	0.023	0.699	3.798	20.889
PZC	0.005	0.152	3.165	17.408
UNIL	0.093	2.812	1.882	10.352
TLW	0.172	5.229	5.270	28.985
AGA	0.117	3.557	3.346	18.403
BOPP	0.195	5.928	4.196	23.078

Table 2 shows the covariance matrix of the stocks. This provides a first-hand information on how the returns move together in a whole.

Table 2. *Covariance Matrix of the Stocks.*

Stock	ETI	GCB	GOIL	TOTAL	FML	GGBL	CLYD	EGL	PZC	UNIL	TLW	AGA	BOPP
ETI	41.770	-2.013	-1.231	-2.284	1.047	0.207	0.362	4.116	-0.910	-1.550	0.355	-0.284	1.350
GCB	-2.013	3.710	-0.047	1.615	-0.479	-0.188	0.027	-2.964	0.426	1.147	-0.095	0.852	-1.269
GOIL	-1.231	-0.047	4.399	-0.166	-0.229	0.412	0.025	0.749	-0.042	-0.015	0.144	-0.015	0.111
TOTAL	-2.284	1.615	-0.166	18.734	-0.646	-0.458	-1.379	-3.483	0.408	1.308	0.020	0.127	-0.174
FML	1.047	-0.479	-0.229	-0.646	4.653	0.799	1.455	1.732	-0.572	-0.545	-0.179	0.757	1.650
GGBL	0.207	-0.188	0.412	-0.458	0.799	2.387	0.446	2.006	0.128	-0.618	0.053	0.642	1.221
CLYD	0.362	0.027	0.025	-1.379	1.455	0.446	21.138	-0.144	0.394	-0.016	-0.425	-0.243	1.232
EGL	4.116	-2.964	0.749	-3.483	1.732	2.006	-0.144	14.427	-1.230	-2.382	0.174	-0.217	3.298
PZC	-0.910	0.426	-0.042	0.408	-0.572	0.128	0.394	-1.230	10.017	1.229	-0.042	0.205	0.168
UNIL	-1.550	1.147	-0.015	1.308	-0.545	-0.618	-0.016	-2.382	1.229	3.542	0.188	0.347	-1.635
TLW	0.355	-0.095	0.144	0.020	-0.179	0.053	-0.425	0.174	-0.042	0.188	27.777	-0.005	-0.504
AGA	-0.284	0.852	-0.015	0.127	0.757	0.642	-0.243	-0.217	0.205	0.347	-0.005	11.199	-0.328
BOPP	1.350	-1.269	0.111	-0.174	1.650	1.221	1.232	3.298	0.168	-1.635	-0.504	-0.328	17.603

Figure 1 shows the monthly plot of risk-return of the stocks. The plot shows that, GOIL, PZC and EGL recorded low returns with higher risk levels compared with FML, AGA, BOPP and TLW which recorded higher returns with somewhat lower risk levels. ETI recorded the highest risk level of 35.547 with an expected return of 2.523. GGBL recorded the least risk with an expected return of 8.498. Since investors are only interested in forming optimal and efficient portfolios, CLYD was not considered since it made a loss. Also, the risk-return plot indicates that, equally weighted portfolio has higher expected return per the level of risk.

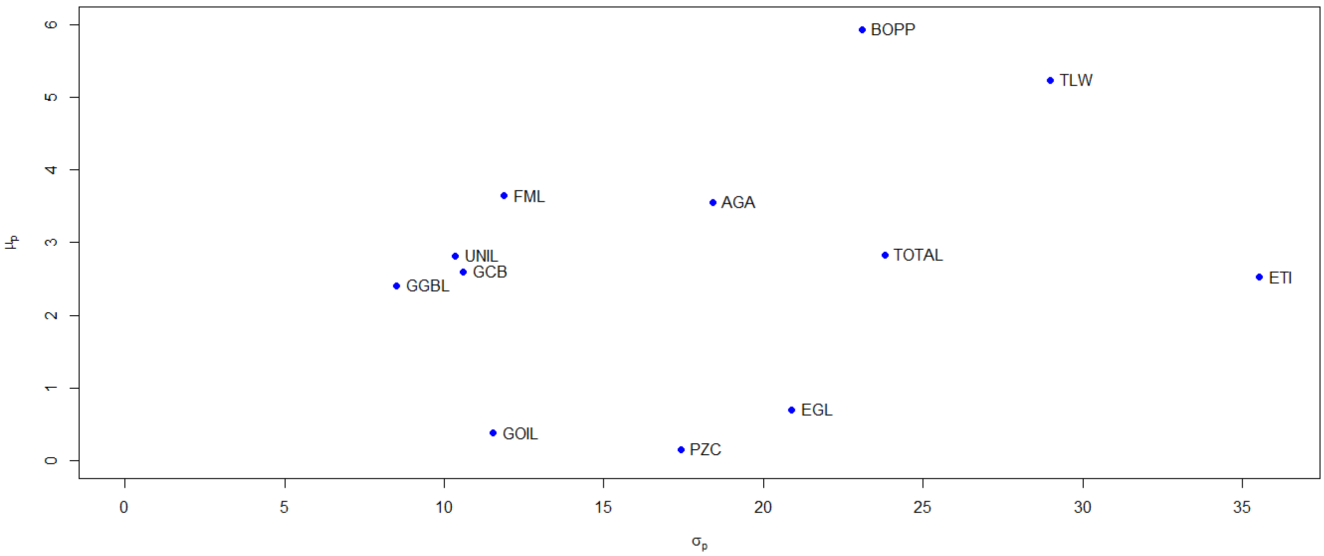

Figure 1. *Monthly plot of the risk and return of the stocks.*

Table 3, shows the estimates of the global minimum-variance portfolio. The results reveals that, the expected return on the portfolio called global minimum-variance is 2.360 and a risk level of 0.766. This means that, there is a 0.766 risk in investing in the minimum-variance portfolio that rewards 2.360. The global minimum-variance portfolio has portfolio weights (asset allocation) as follows;

$$m_{eti} = 0.014, m_{gcb} = 0.166, m_{goil} = 0.139, m_{total} = 0.020, m_{fml} = 0.150,$$

$$m_{ggbl} = 0.188, m_{clyd} = 0.014, m_{egl} = 0.024, m_{pzc} = 0.061, m_{unil} = 0.163,$$

$$m_{tlw} = 0.020, m_{aga} = 0.014\ and m_{bopp} = 0.026.$$

In a vector form, the allocation of assets for the global minimum-variance portfolio is given by;

$$m = (0.014,0.166,0.139,0.020\ 0.150,0.188,0.014,0.024,0.061,0.163,0.020,0.014,0.026)' \tag{32}$$

In other to achieve this return, an investor needs to allocate assets given Equation 32 for investing in the portfolio with the least risk.

Table 3. *Global Minimum Variance Portfolio.*

Stock	Global Minimum Variance portfolio Weight	Expected Return ($\mu_{p,m}$)	Portfolio Std. dev ($\sigma_{p,m}$)
ETI	0.014	2.360	0.766
GCB	0.166		
GOIL	0.139		
TOTAL	0.020		
FML	0.150		
GGBL	0.188		
CLYD	0.014		
EGL	0.024		
PZC	0.061		
UNIL	0.163		
TLW	0.020		
AGA	0.014		
BOPP	0.026		

Table 4, shows the efficient portfolio with the same expected return as a given stock. The results indicate that, when ETI, GCB, GOIL, TOTAL, FML, GGBL, EGL, PZC, UNIL, TLW, AGA and BOPP with expected returns 2.523, 2.593, 0.377, 2.827, 3.642, 2.405, 0.699, 0.152, 2.812, 5.229, 3.559 and 5.928 respectively, then in other to have a portfolio whose expected return will be the same as that of as any of the above assets, an investor need to bear a risk 0.771, 0.776, 1.288, 0.805, 1.019, 0.067, 1.157, 1.384, 0.802, 1.685, 0.990 and 2.017 for holding portfolio 1, 2,…,12 respectively which are all lesser than the individual risk associated with the assets. This indicates that, one reduces risk by diversifying in several assets that are uncorrelated. In portfolio 1, 2, …, 12, the highest proportion of asset allocation is found in GGBL (0.188), GGBL (0.189), GGBL (0.188), GOIL (0.382), GGBL (0.190), GGBL (0.194), GGBL(0.179), PZC (0.192), GGBL(0.190), FML (0.356), FML (0.236) and FML (0.406) respectively. GGBL is seen as the stock with the highest allocation of wealth in most of the portfolios. This is so because from Table 1, GGBL exhibited the least standard deviation (risk) and since investors are interested in minimizing risk given a target expected return, hence much wealth allocation in GGBL. The least allocation of wealth in portfolio 1, 2, …, 12 was CLYD (0.007), CLYD (0.004), BOPP (-0.102), CLYD (-0.005), CLYD (-0.040), CLYD (0.012), BOPP (-0.081), BOPP (-0.116), CLYD (-0.005), GOIL (-0.212), EGL (-0.177) and GOIL (-0.297) respectively. Six out of the 12 portfolios had CLYD exhibiting the least asset allocation. This is because even though from Table 1, CLYD had a higher risk level but no compensation for holding it since it made a loss. For an investor to adequately minimize risk in other to achieve the expected return in each portfolio, the proportion of wealth to be allocated to each asset in each portfolio is given by the following vectors;

$$w_{portfolio\ 1} = (0.014, 0.169, 0.119, 0.021, 0.162, 0.188, 0.007, 0.020, 0.052, 0.169, 0.025, 0.017, 0.037)' \quad (33)$$

$$w_{portfolio\ 2} = (0.014, 0.0.170, 0.111, 0.021, 0.167, 0.189, 0.004, 0.017, 0.047, 0.172, 0.027, 0.019, 0.041)' \quad (34)$$

$$w_{portfolio\ 3} = (0.119, 0.130, 0.382, 0.011, 0.008, 0.178, 0.098, 0.084, 0.179, 0.084, -0.036, -0.028, -0.102)' \quad (35)$$

$$w_{portfolio\ 4} = (0.146, 0.174, 0.082, 0.022, 0.183, 0.190, -0.005, 0.010, 0.033, 0.181, 0.034, 0.024, 0.056)' \quad (36)$$

$$w_{portfolio\ 5} = (0.155, 0.189, -0.017, 0.026, 0.242, 0.194, -0.040, -0.014, -0.015, 0.213, 0.057, 0.041, 0.109)' \quad (37)$$

$$w_{portfolio\ 6} = (0.014, 0.167, 0.134, 0.020, 0.153, 0.189, 0.012, 0.023, 0.059, 0.164, 0.022, 0.015, 0.029)' \quad (38)$$

$$w_{portfolio\ 7} = (0.012, 0.136, 0.342, 0.013, 0.031, 0.179, 0.084, 0.075, 0.160, 0.097, -0.027, -0.021, -0.081)' \quad (39)$$

$$w_{portfolio\ 8} = (0.012, 0.126, 0.409, 0.010, -0.008, 0.177, 0.107, 0.091, 0.192, 0.075, -0.043, -0.032, -0.116)' \quad (40)$$

$$w_{portfolio\ 9} = (0.015, 0.172, 0.084, 0.022, 0.182, 0.190, -0.005, 0.011, 0.034, 0.180, 0.033, 0.023, 0.055)' \quad (41)$$

$$w_{portfolio\ 10} = (0.017, 0.218, -0.212, 0.033, 0.356, 0.202, -0.107, -0.062, -0.109, 0.276, 0.102, 0.074, 0.211)' \quad (42)$$

$$w_{portfolio\ 11} = (0.015, 0.188, -0.007, 0.025, 0.236, 0.194, -0.036, -0.117, -0.010, 0.210, 0.055, 0.039, 0.103)' \quad (43)$$

$$w_{portfolio\ 12} = (0.018, 0.231, -0.297, 0.036, 0.406, 0.205, -0.136, -0.083, -0.151, 0.304, 0.122, 0.089, 0.256)' \quad (44)$$

The efficient portfolios were selected by taking into consideration the expected return of each portfolio. That is, any portfolio with expected return greater or equal the expected return of the global minimum-variance portfolio is considered efficient portfolio otherwise the portfolio is an inefficient one. From the results, portfolios 1, 2, 4, 5, 6, 9, 10, 11 and 12 with expected return 2.523, 2.593, 2.827, 3.642, 2.405, 2.812, 5.229, 3.559 and 5.928 respectively are considered efficient portfolios since their expected return each is greater than the expected return of the global minimum variance portfolio. This indicates that, these portfolios have maximum expected for the level risk estimated. Portfolios 3, 7 and 8 with expected return 0.377, 0.699 and 0.152 respectively are considered inefficient portfolios since their expected each is less than the expected return of the global minimum-variance portfolio.

Table 4. *Efficient Portfolio with the same expected return as a given stock.*

Portfolio No.	Stock	Stock Expected Return	Weight	Portfolio Expected Return ($\mu_{p,o}$)	Portfolio Std. dev ($\sigma_{p,o}$)
1	ETI*	2.523	0.014	2.523	0.771
	GCB		0.169		
	GOIL		0.119		
	TOTAL		0.021		
	FML		0.162		
	GGBL		0.188		
	CLYD		0.007		
	EGL		0.020		
	PZC		0.052		
	UNIL		0.169		
	TLW		0.025		
	AGA		0.017		
	BOPP		0.037		
	ETI		0.014		
2	GCB*	2.593	0.170	2.593	0.776
	GOIL		0.111		
	TOTAL		0.021		
	FML		0.167		
	GGBL		0.189		
	CLYD		0.004		
	EGL		0.017		
	PZC		0.047		
	UNIL		0.172		
	TLW		0.027		
	AGA		0.019		
	BOPP		0.041		
	ETI		0.119		
	GCB		0.130		
3	GOIL*	0.377	0.382	0.377	1.288
	TOTAL		0.011		
	FML		0.008		
	GGBL		0.178		
	CLYD		0.098		
	EGL		0.084		
	PZC		0.179		
	UNIL		0.084		
	TLW		-0.036		
	AGA		-0.028		
	BOPP		-0.102		
	ETI		0.146		
	GCB		0.174		
	GOIL		0.082		
4	TOTAL*	2.827	0.022	2.827	0.805
	FML		0.183		
	GGBL		0.190		
	CLYD		-0.005		
	EGL		0.010		
	PZC		0.033		
	UNIL		0.181		
	TLW		0.034		
	AGA		0.024		
	BOPP		0.056		
	ETI		0.155		
	GCB		0.189		
	GOIL		-0.017		
	TOTAL		0.026		
5	FML*	3.642	0.242	3.642	1.019
	GGBL		0.194		
	CLYD		-0.040		

Portfolio No.	Stock	Stock Expected Return	Weight	Portfolio Expected Return ($\mu_{p,o}$)	Portfolio Std. dev ($\sigma_{p,o}$)
	EGL		-0.014		
	PZC		-0.015		
	UNIL		0.213		
	TLW		0.057		
	AGA		0.041		
	BOPP		0.109		
	ETI		0.014		
	GCB		0.167		
	GOIL		0.134		
	TOTAL		0.020		
	FML		0.153		
6	GGBL*	2.405	0.189	2.405	0.767
	CLYD		0.012		
	EGL		0.023		
	PZC		0.059		
	UNIL		0.164		
	TLW		0.022		
	AGA		0.015		
	BOPP		0.029		
	ETI		0.012		
	GCB		0.136		
	GOIL		0.342		
	TOTAL		0.013		
	FML		0.031		
	GGBL		0.179		
	CLYD		0.084		
7	EGL*	0.699	0.075	0.699	1.157
	PZC		0.160		
	UNIL		0.097		
	TLW		-0.027		
	AGA		-0.021		
	BOPP		-0.081		
	ETI		0.012		
	GCB		0.126		
	GOIL		0.409		
	TOTAL		0.010		
	FML		-0.008		
	GGBL		0.177		
	CLYD		0.107		
	EGL		0.091		
8	PZC*	0.152	0.192	0.152	1.384
	UNIL		0.075		
	TLW		-0.043		
	AGA		-0.032		
	BOPP		-0.116		
	ETI		0.015		
	GCB		0.174		
	GOIL		0.084		
	TOTAL		0.022		
	FML		0.182		
	GGBL		0.190		
	CLYD		-0.005		
	EGL		0.011		
	PZC		0.034		
9	UNIL*	2.812	0.180	2.812	0.802
	TLW		0.033		
	AGA		0.023		
	BOPP		0.055		
	ETI		0.017		
	GCB		0.218		
	GOIL		-0.212		
	TOTAL		0.033		
	FML		0.356		
	GGBL		0.202		
	CLYD		-0.107		
	EGL		-0.062		
	PZC		-0.109		
	UNIL		0.276		
10	TLW*	5.229	0.102	5.229	1.685
	AGA		0.074		
	BOPP		0.211		
	ETI		0.015		
	GCB		0.188		

Portfolio No.	Stock	Stock Expected Return	Weight	Portfolio Expected Return ($\mu_{p,o}$)	Portfolio Std. dev ($\sigma_{p,o}$)
	GOIL		-0.007		
	TOTAL		0.025		
	FML		0.236		
	GGBL		0.194		
	CLYD		-0.036		
	EGL		-0.117		
	PZC		-0.010		
	UNIL		0.210		
	TLW		0.055		
11	AGA*	3.559	0.039	3.559	0.990
	BOPP		0.103		
	ETI		0.018		
	GCB		0.231		
	GOIL		-0.297		
	TOTAL		0.036		
	FML		0.406		
	GGBL		0.205		
	CLYD		-0.136		
	EGL		-0.083		
	PZC		-0.151		
	UNIL		0.304		
	TLW		0.122		
	AGA		0.089		
12	BOPP*	5.928	0.256	5.928	2.017

* Given stock

Figure 2, shows the efficient frontier of the portfolios under consideration. It can be seen that, all the portfolios are optimal since they are all on the minimum-variance frontier. Also, the efficient portfolios (ETI, GCB, TOTAL, FML, GGBL, CLYD, UNIL, TLW, AGA and BOPP) are located in the upper portion-at or above the global minimum-variance portfolio (Global minimum) whereas the inefficient portfolios (GOIL, PZC and EGL) are found beneath the Global minimum.

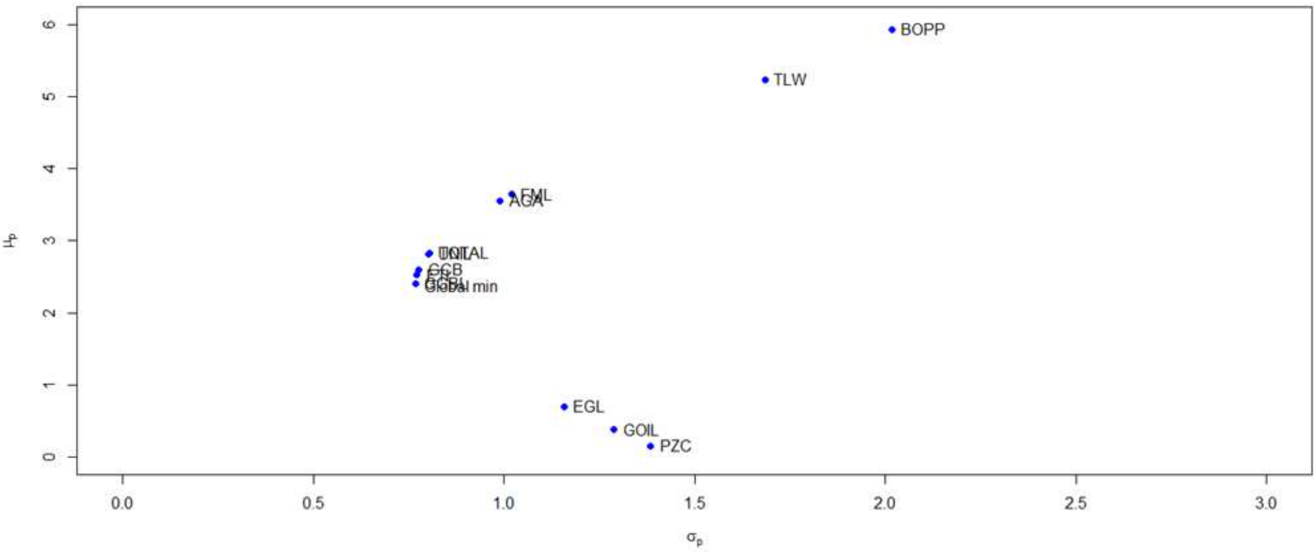

Figure 2. *Efficient Frontier.*

4. Conclusion

The purpose of this paper is to construct optimal portfolio using matrix approach. The results indicate that, all the portfolios are optimal and that portfolios 1, 2, 4, 5, 6, 9, 10, 11 and 12 with expected return 2.523, 2.593, 2.827, 3.642, 2.405, 2.812, 5.229, 3.559 and 5.928 respectively are efficient portfolios whereas portfolios 3, 7 and 8 with expected return 0.377, 0.699 and 0.152 respectively are inefficient portfolios with reference to the expected return of the global minimum variance portfolio (2.360). GGBL is seen as the stock with the highest allocation of wealth in most of the portfolios. Six out of the 12 portfolios had CLYD exhibiting the least asset allocation. It is therefore advisable for investors to consider investing in the efficient portfolios by taking into consideration the weight of each portfolio so as to minimize risk in other to get the desired return. Also, it is seen that investing in only one asset bares a higher risk than investing in several assets hence the need for investors to diversify their portfolios.

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